# Exact Analytical Solution for the Via-Plate Capacitance in Multiple-Layer Structures


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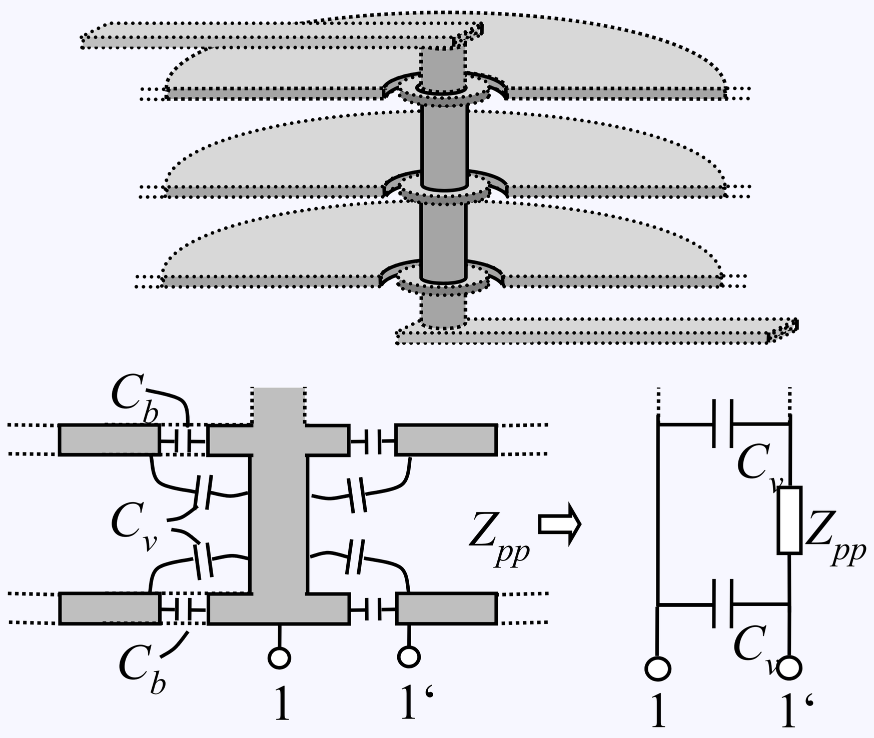
## INTRODUCTION

Via interconnections are numerously used on multilayer printed-circuit boards

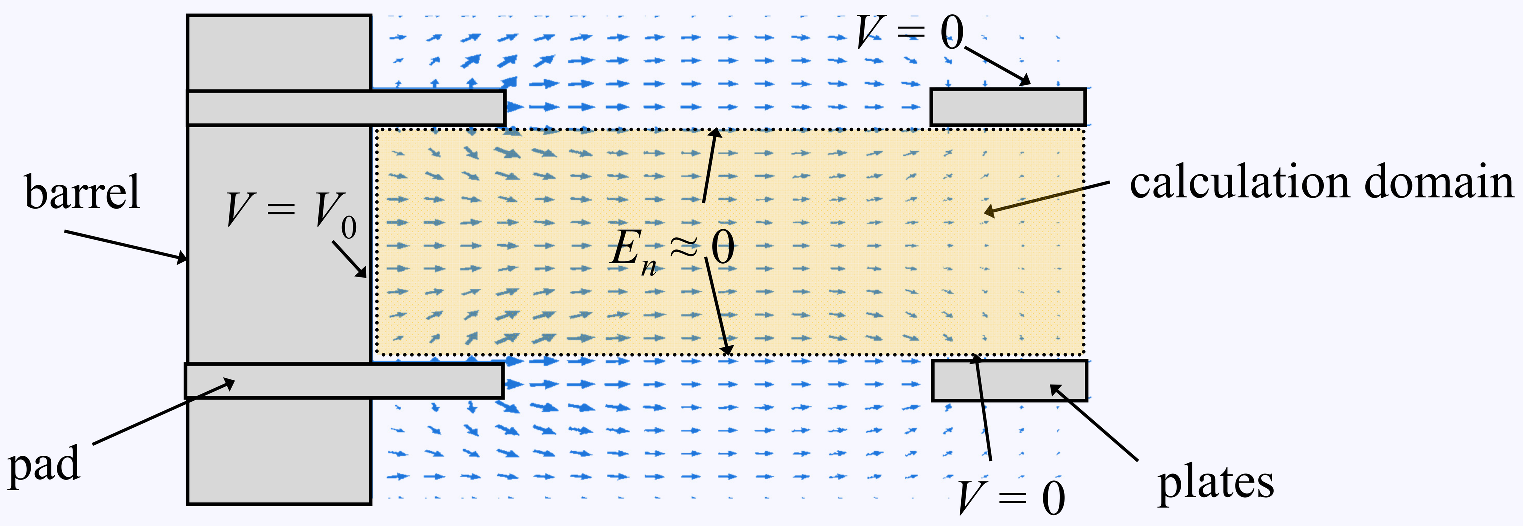
- Supply or ground planes represet a resonating cavity
- High-speed signals excite cavity, due to the capacitive coupling between the via and the plates

⇨ signal-integrity (SI) issues and electromagnetic interference (EMI)



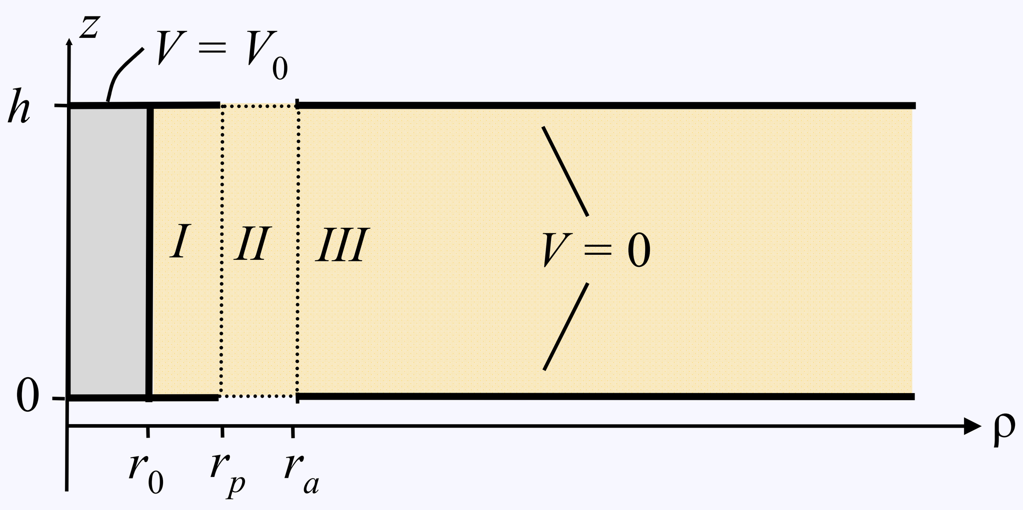

### *Field distribution from numerical full-wave simulation*

- Calculation domain is electrical small ⇒ quasistatic analysis
- Boundary value problem Dirichlet and Neumann conditions




## ANALYTICAL SOLUTION FOR ELECTRIC POTENTIAL

Calculation domain, divided into three sections with uniform boundary condition




2D Laplace equation for cylindrically symmetric problem:

$$\Delta V(\rho,z)=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)+\frac{\partial^2 V}{\partial z^2}=0$$

General solution of Laplace's equation:

$$R(\rho)=\begin{cases}A_0+B_0\cdot\ln(\rho/\rho_0) & k_z=0\\ A\cdot I_0(k_z\rho)+B\cdot K_0(k_z\rho) & k_z\neq 0\end{cases}$$

$$Z(z)=\begin{cases}C_0+D_0\cdot z & k_z=0\\ C\cdot\cos(k_z z)+D\cdot\sin(k_z z) & k_z\neq 0\end{cases}$$

Subdomain solutions:

$$V_1(\rho,z)=V_0-\sum_{m=1}^{\infty}A_m\frac{K_0(k_{z,1}^m\rho)I_0(k_{z,1}^m r_0)-K_0(k_{z,1}^m r_0)I_0(k_{z,1}^m\rho)}{K_0(k_{z,1}^m r_p)I_0(k_{z,1}^m r_0)-K_0(k_{z,1}^m r_0)I_0(k_{z,1}^m r_p)}\cdot\sin(k_{z,1}^m z)$$

$$V_2(\rho,z)=B_0\cdot(\ln(\rho/r_0)/\ln(r_a/r_0))+C_0+\sum_{m=1}^{\infty}\left(B_m\cdot\frac{I_0(k_{z,2}^m\rho)}{I_0(k_{z,2}^m r_a)}+C_m\cdot\frac{K_0(k_{z,2}^m\rho)}{K_0(k_{z,2}^m r_p)}\right)\cdot\cos(k_{z,2}^m z)$$

$$V_3(\rho,z)=\sum_{m=1}^{\infty}D_m\cdot K_0(k_{z,3}^m\rho)\cdot\sin(k_{z,3}^m z)$$

Linear equation system for the *p*-th set of equation as obtained from the boundary conditions

$$A_p=\sum_{m=1}^{\infty}B_m\alpha_{pm}^{AB}+C_m\alpha_{pm}^{AC}\qquad B_p=C_p\alpha_p^{BC}+\sum_{m=0}^{\infty}D_m\alpha_{pm}^{BD}$$

$$C_p=V_0\delta_{p,0}+B_p\alpha_p^{CB}+\sum_{m=0}^{\infty}A_m\alpha_{pm}^{CA}\qquad D_p=\sum_{m=1}^{\infty}B_m\alpha_{pm}^{DB}+C_m\alpha_{pm}^{DC}$$

The sums are truncated after *M* terms yielding a matrix equation system

$$\mathbf{A}\cdot\mathbf{U}=\mathbf{V}$$

*Coefficient matrix*

$$\mathbf{A}=[\alpha]$$

*Excitation and solution vector*

$$\mathbf{U}=(A_1\quad A_2\quad\cdots\quad A_M\quad\cdots\quad B_m\quad\cdots\quad C_m\quad\cdots\quad D_M)$$

$$\mathbf{V}=(0\quad 0\quad\cdots\quad V_0\quad\cdots\quad 0)$$

Matrix elements $\alpha_{pm}$ are calculated by exact analytical expressions

## CAPACITANCE CALCULATION

### *Calculation by Gauss Law*

Electric flux density

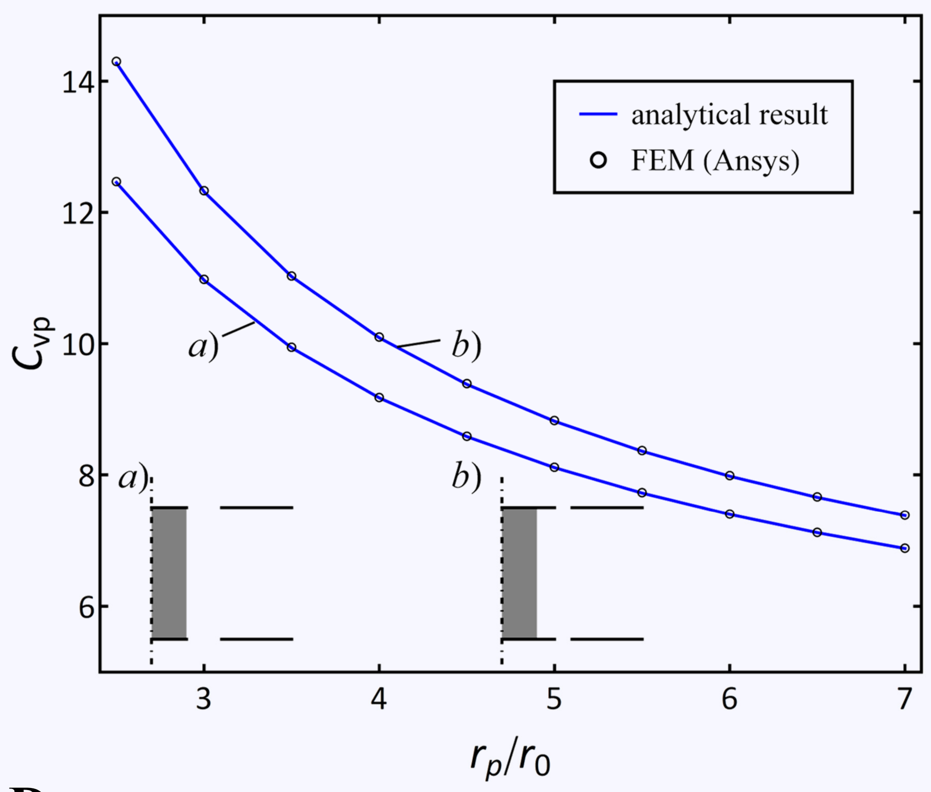
$$D_\rho(\rho=r_p,z)=-\varepsilon\cdot\frac{\partial}{\partial\rho}V_2(\rho,z)\Big|_{\rho=r_p}$$

Capacitance

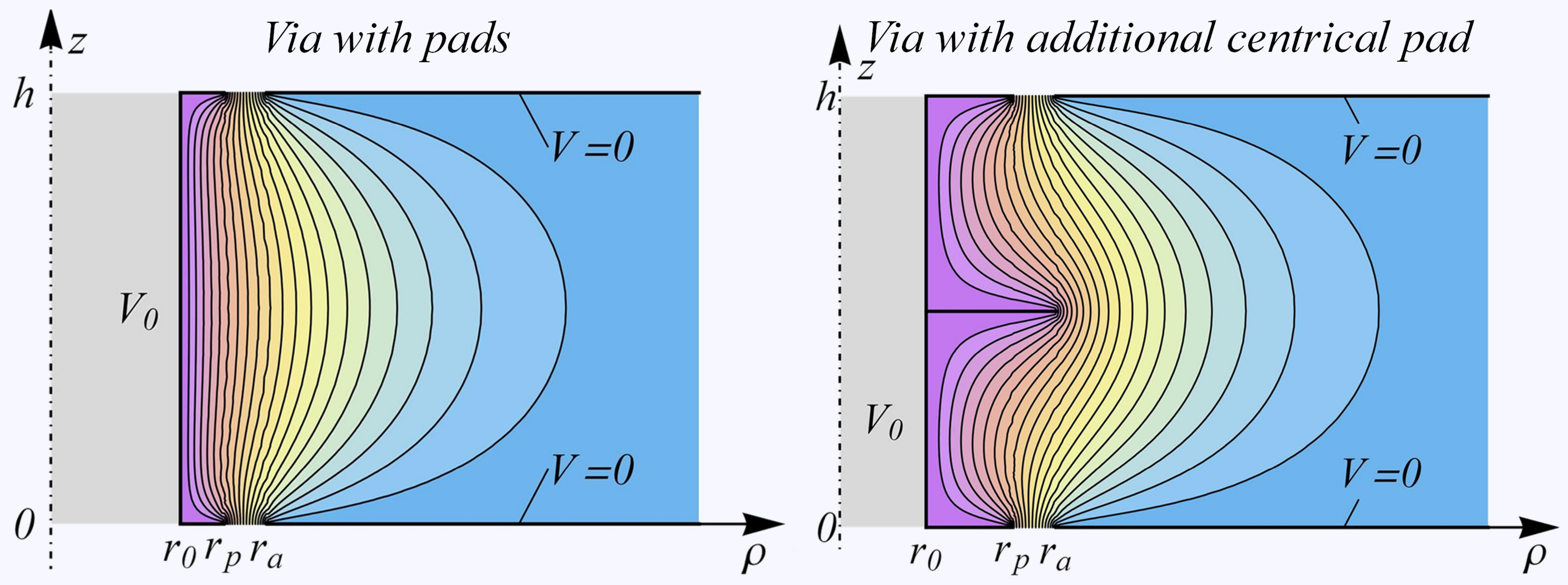
$$C_{pp}=\frac{Q}{V_0}=\frac{1}{V_0}\oint\!\!\oint_{A_{via}}D_\rho(\rho=r_p,z)\,\mathrm{d}A$$

$$\Rightarrow C_{pp}=\frac{2\pi\varepsilon h B_0}{\ln r_a/r_0}$$

### *Comparison with FEM simulation*

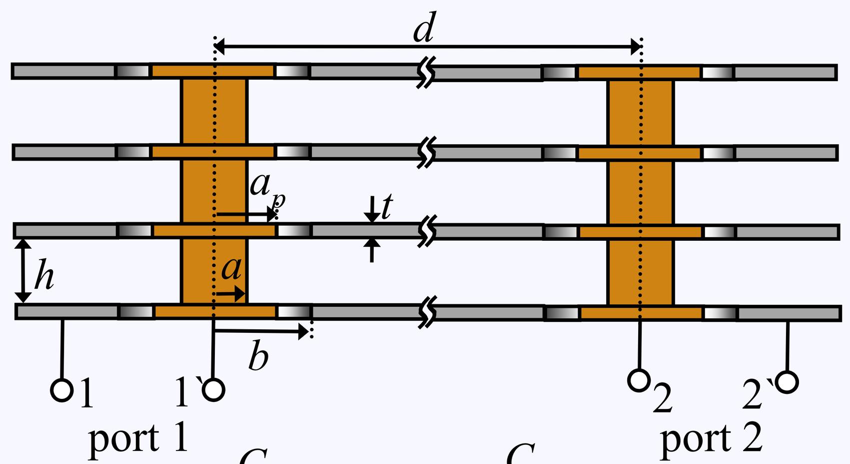


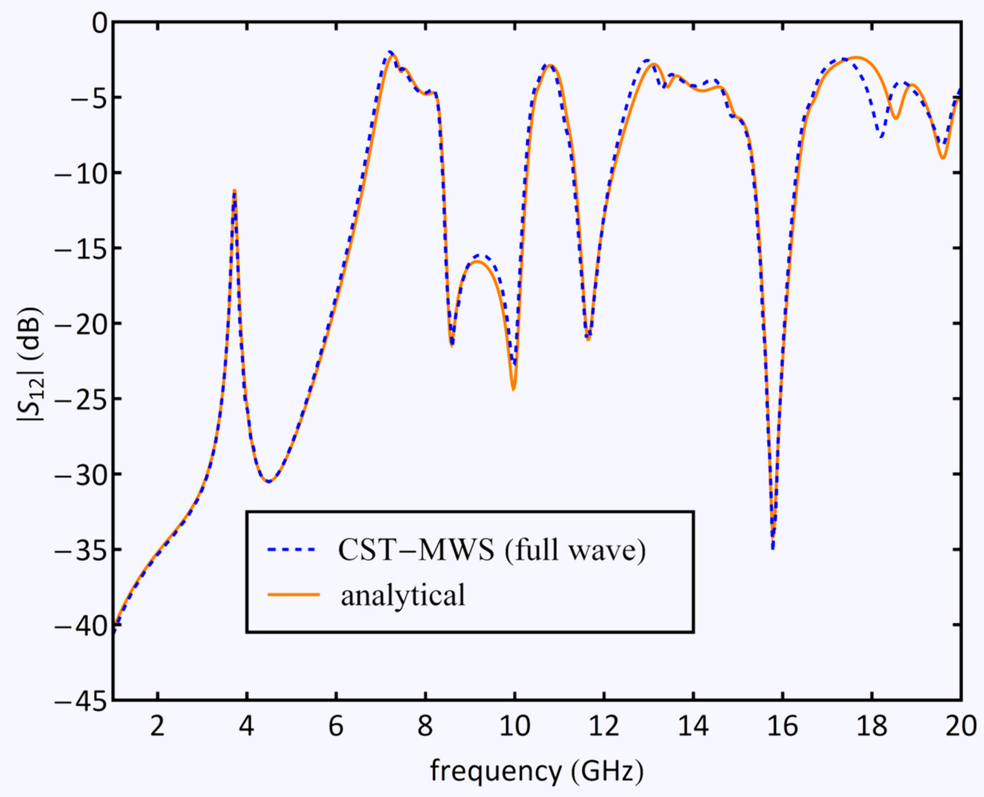

## POTENTIAL DISTRIBUTION

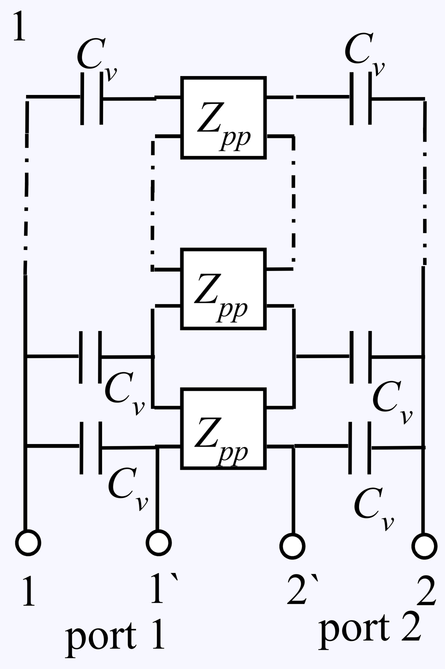



## FULL-WAVE COMPUTATIONAL EXAMPLE

- 4-Layer PCB with two vias
- Network-model in good agreement with full-wave simulation




Network Model