# Lehrstuhl Theoretische Elektrotechnik

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# Inductive Network Model for the Radiation Analysis of Electrically Small Parallel-Plate Structures


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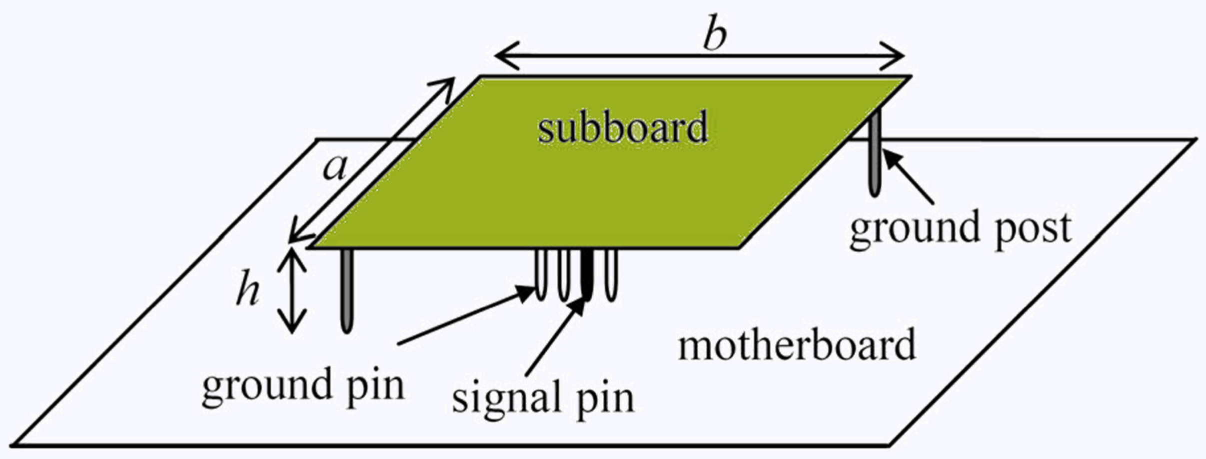
## Introduction

Vertical interconnections between electrically small structures as often encountered in motherboard subboard applications are source of radiated emissions

Modeling approach:

- equivalent circuit for interconnections
- hertzian-dipole radiation characteristic

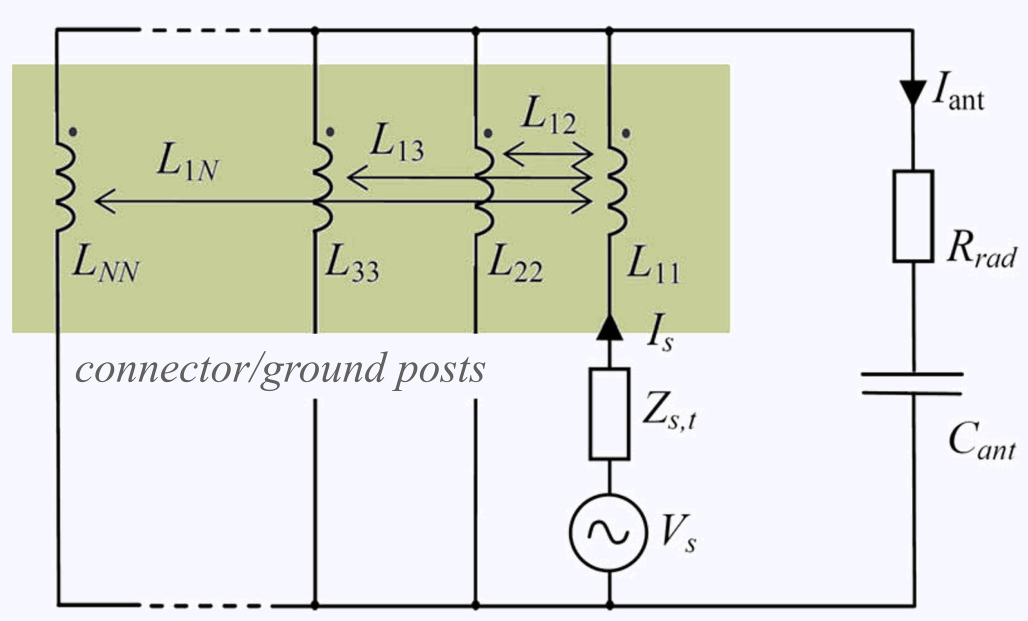



Electrical small parallel-plate structure ( $\lambda >> a, b >> h$)

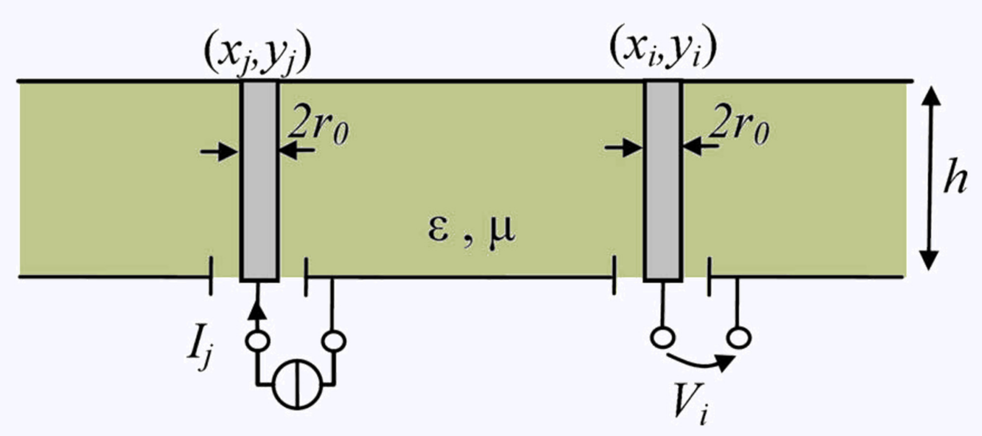
## Quasistatic Greens Function

$$G(r|r_s) = j\omega \frac{\mu h}{ab} \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{K_{m,n}}{k_{xm}^2 + k_{yn}^2} \times \cos(k_{xm}x)\cos(k_{yn}y)\cos(k_{xm}x_s)\cos(k_{yn}y_s)$$

## Low frequency equivalent-circuit Model




## Network elements




Side view of source and observation ports with circular geometry and radius $r_0$

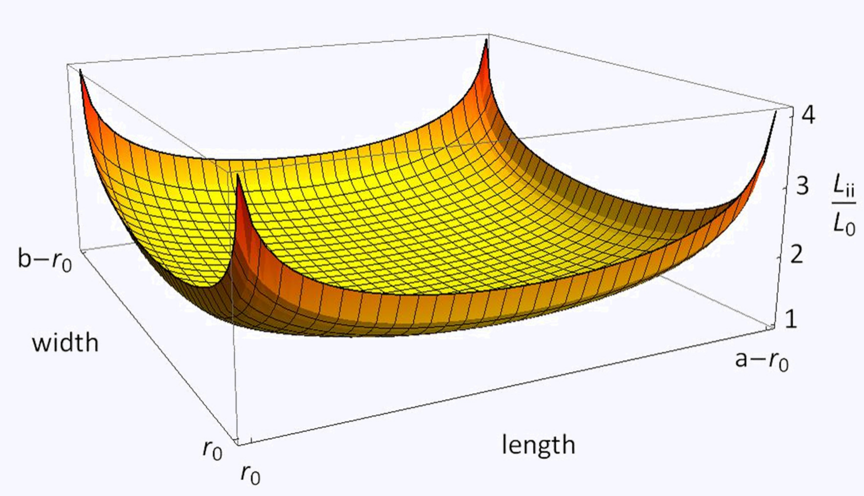
### *Self-inductance*

$$L_{ii} = -\frac{h\mu}{4\pi}\left( \begin{array}{l} \ln(\cosh(2\pi y_i/a) - \cos(2\pi x_i/a)) \\ +\ln(\cosh(2\pi(b-y_i)/a) - \cos(2\pi x_i/a)) \\ +2\ln(\sinh(\pi y_i/a)\sinh(\pi(b-y_i)/a)) \\ +2\ln(\sin(\pi x_i/a)) + 2\ln(\pi r_0/a) \\ +8\ln(2) - \frac{b}{a}\frac{13}{3}\pi - 4\pi\frac{(y_i-b/2)^2}{ba} \end{array} \right)$$

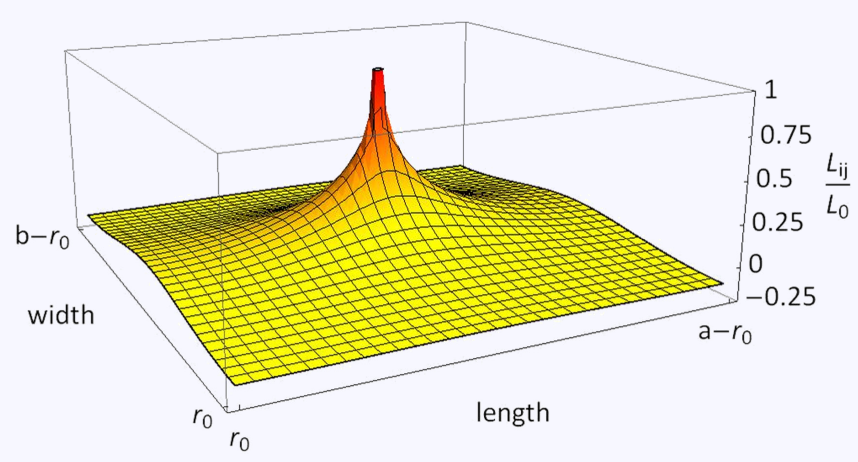
### *Mutual-inductance*

$$L_{ij} = -\frac{\mu h}{4\pi}\left( \begin{array}{l} W(y_i-y_j,x_i,x_j) + W(2b-(y_i+y_j),x_i,x_j) \\ +W(2b-(y_i-y_j),x_i,x_j) + W(y_i+y_j,x_i,x_j) \end{array} \right) + \mu h\left(\frac{y_i^2+y_j^2}{2ab} - \frac{y_i}{a}\right) - \frac{\mu h}{\pi}\ln(4) + \mu h\frac{b}{a}\frac{7}{3}$$

with $$W(y',x_i,x_j) = \ln\left( \begin{array}{l} (\cosh(\pi y'/a) - \cos((x_i+x_j)\pi/a)) \\ \times(\cosh(\pi y'/a) - \cos((x_i-x_j)\pi/a)) \end{array} \right)$$




Normalized self inductance, depending on the port's position




Normalized mutual inductance to a port in the plates center

### *Plate capacitance*

$$C_{ant} = \varepsilon\frac{ab}{h}\left(1 + \frac{2h}{\pi a}\left(1 + \ln\left(\frac{\pi a}{h}\right)\right)\right) \times \left(1 + \frac{2h}{\pi b}\left(1 + \ln\left(\frac{\pi b}{h}\right)\right)\right)$$

### *Radiation resistance*

Hertzian dipole above infinite plane

$$R_{rad} = 40\pi^2\left(\frac{2h}{c_0}f\right)^2$$

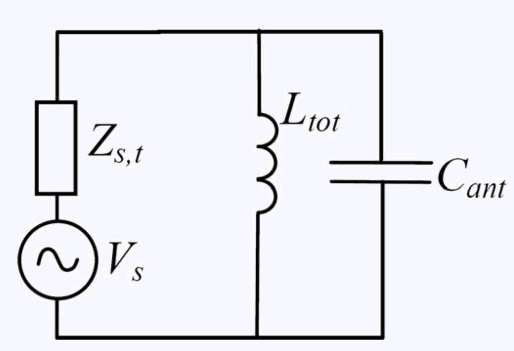
## Resonance Frequency estimation

Parallel resonance of signal return path:

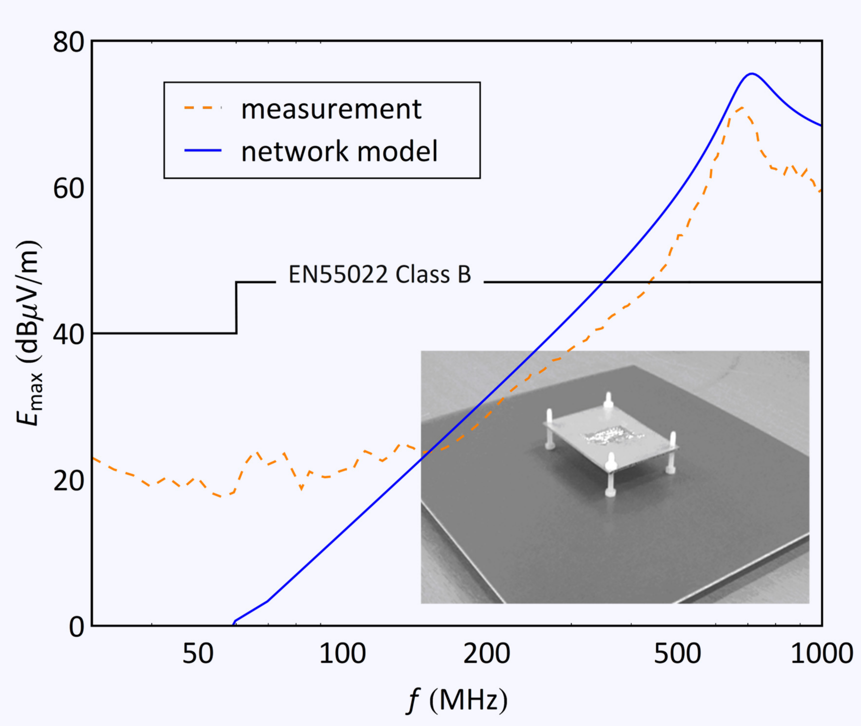
$$f_{res} \approx \frac{1}{2\pi\sqrt{L_{tot}C_{ant}}}.$$

Parallel connection of all signal return inductances: $$L_{tot} = \frac{1}{\sum_{ij} L_{ij}^{-1}}$$

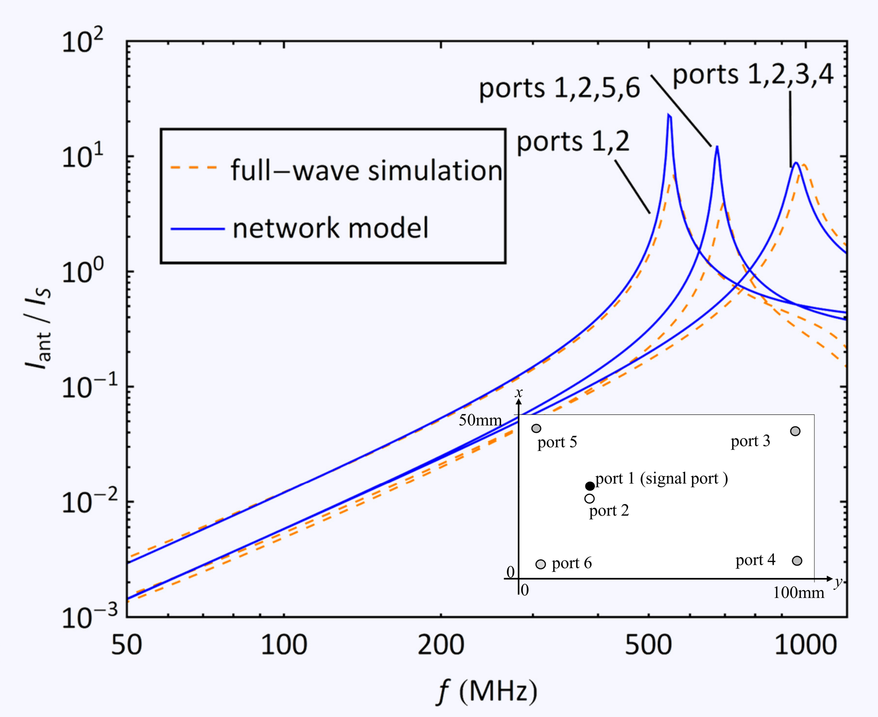
Reduced resonant network




## Results and conclusion




Frequency response of the maximum radiated field strength




Frequency response of the common-mode (antenna) current for different combinations

- Validation of network model by full-wave simulation and measurement
- Resonance frequency can be estimated in advance
- Accurate analytical expressions for equivalent-circuit elements provide physical insight and low computational effort